Constant-Length Labelling Schemes for Deterministic Radio Broadcast

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RADIO NETWORK

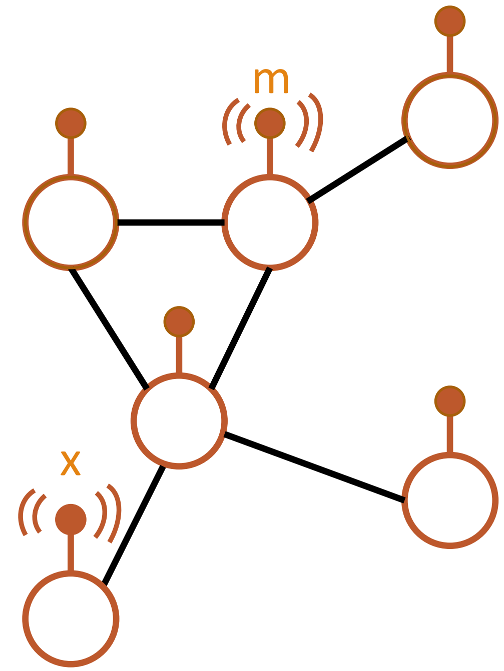
Represented by an undirected graph

- n = number of nodes

Synchronous: time is divided into slots

In each slot, a node can either:

- transmit a message or
- listen

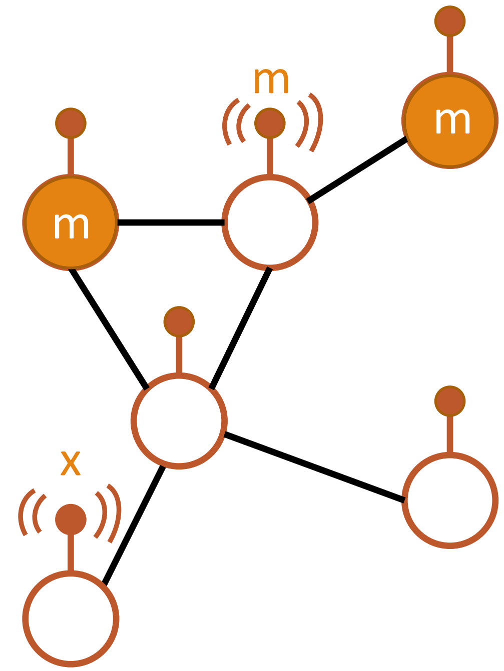


RADIO NETWORK

A node receives a message in a slot

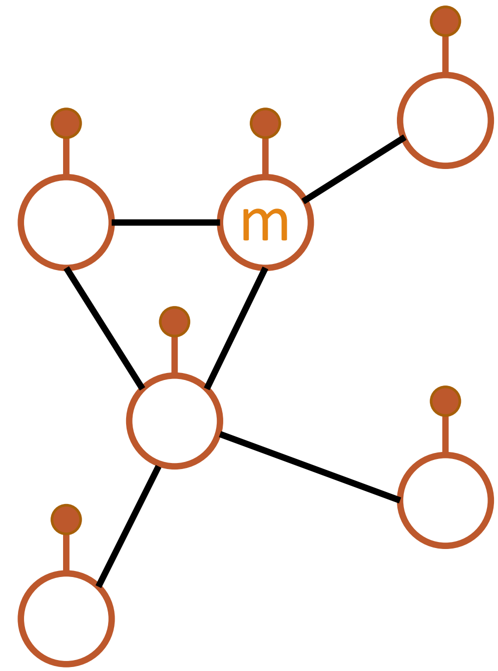
- if it is listening and
- exactly one of its neighbours transmits.

If 2 or more neighbours of a node transmit, there is a collision and the node receives nothing.



RADIO BROADCAST PROBLEM

- Initially, the source knows a source message m.
- Goal: all nodes eventually know m.



DETERMINISTIC RADIO BROADCAST

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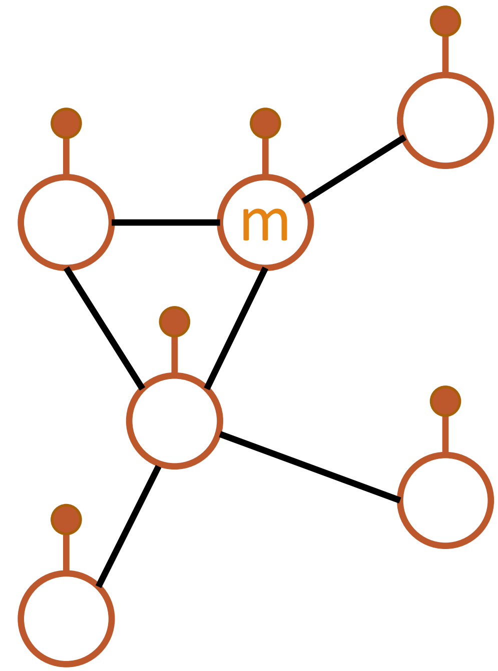
The same deterministic algorithm is used by each node to determine when it transmits.



DETERMINISTIC RADIO BROADCAST

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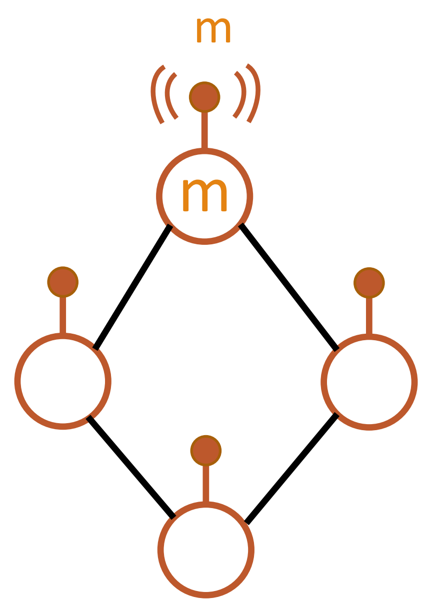
The same deterministic algorithm is used by each node to determine when it transmits.

- If nodes have no labels, there are networks in which deterministic radio broadcast is impossible.

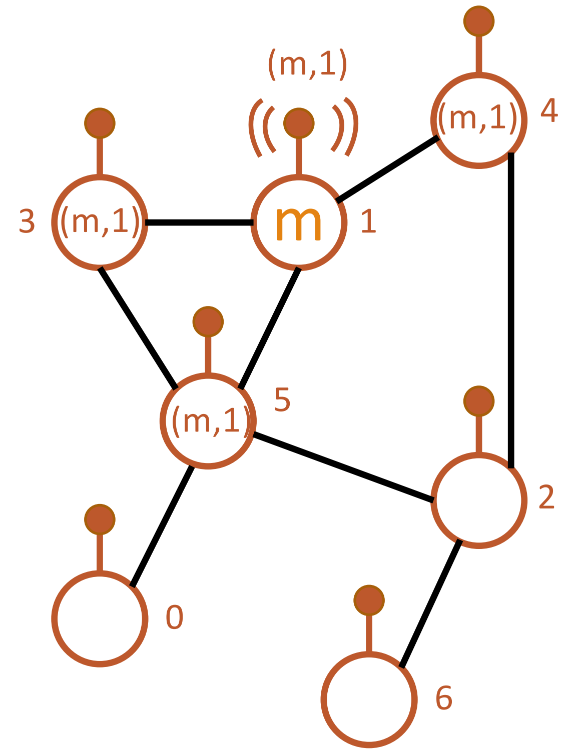


ROUND-ROBIN ALGORITHM

If each node has a distinct label from {0,…,n-1}
then a round-robin algorithm can be used.

The source transmits (m,1).

When a non-source node with label L node
first receives a message, say (m,t),
it sets its counter to t, which it increments each slot.
When its counter mod n is first equal to L,
it transmits (m,L).



slot 1

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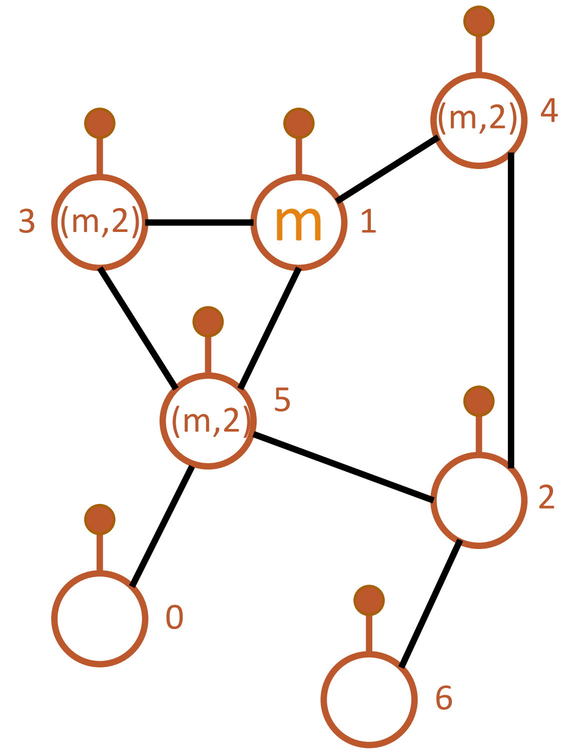

slot 2

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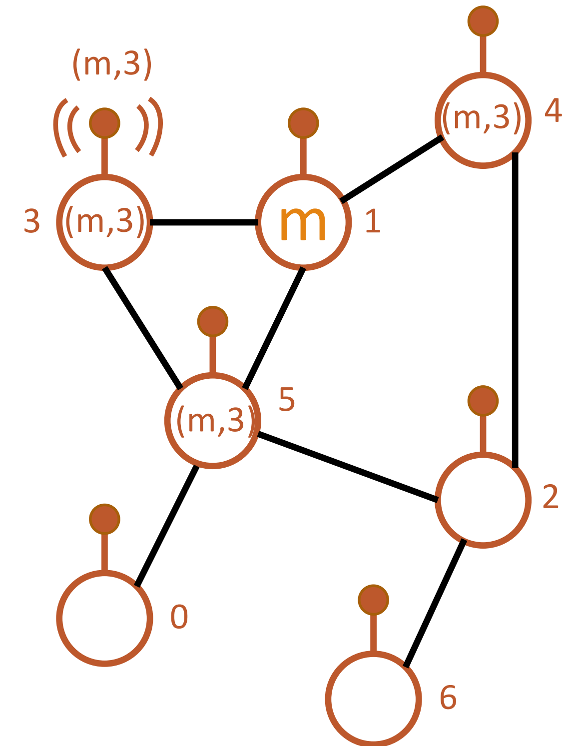

slot 3

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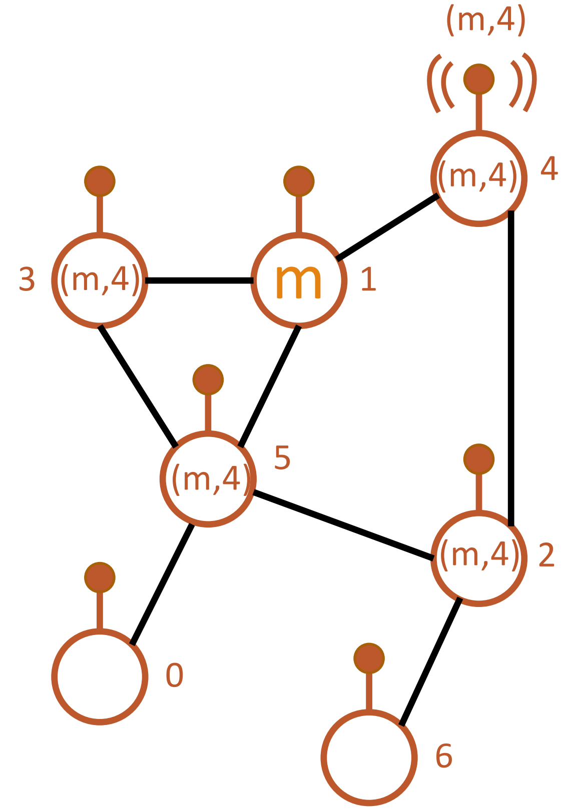

slot 4

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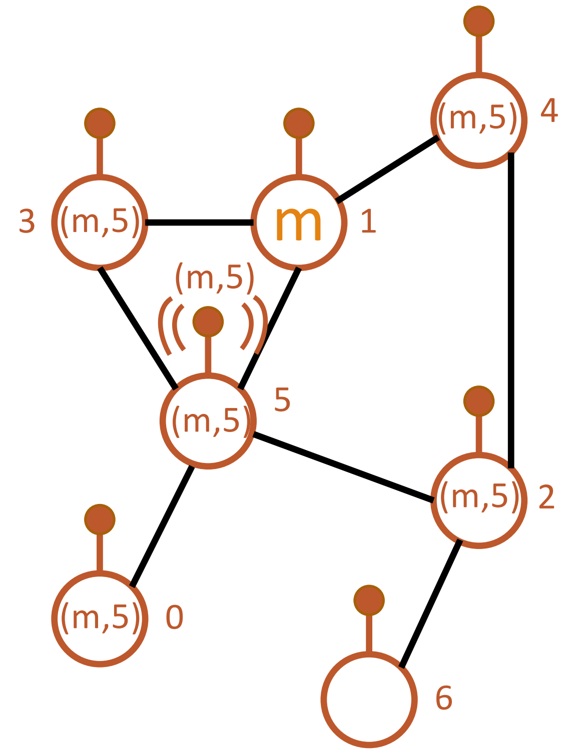

slot 5

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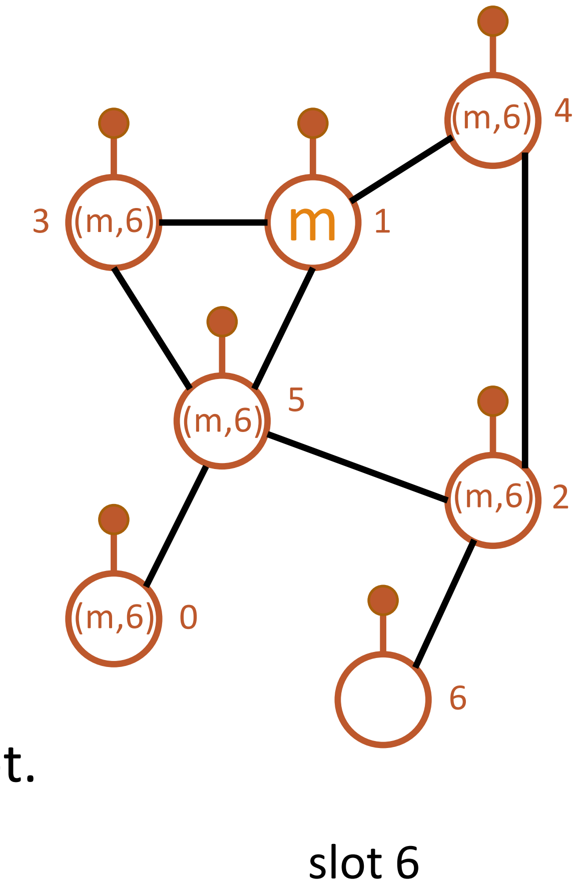

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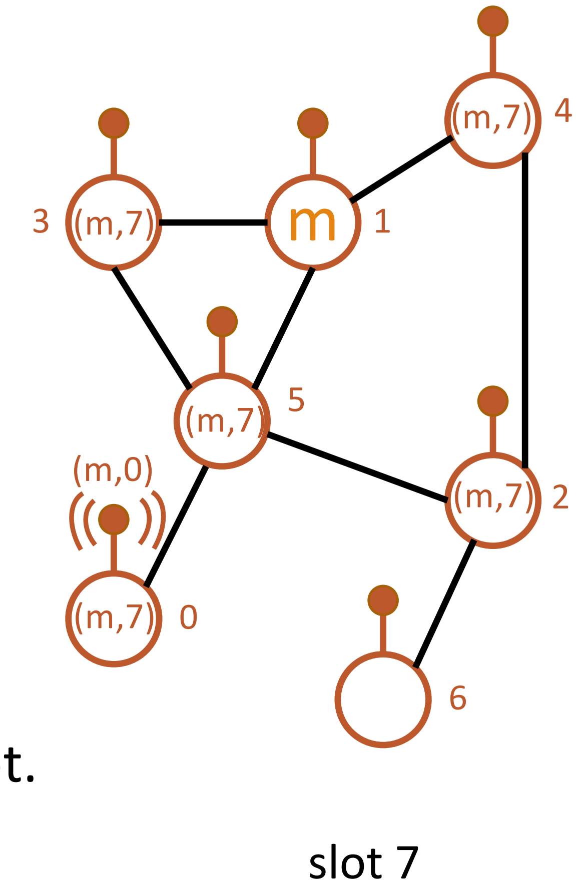

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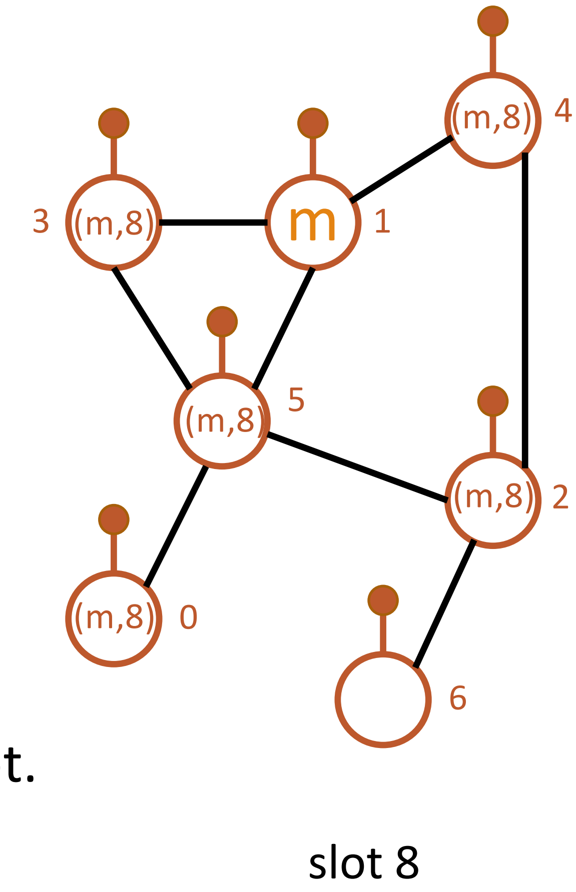

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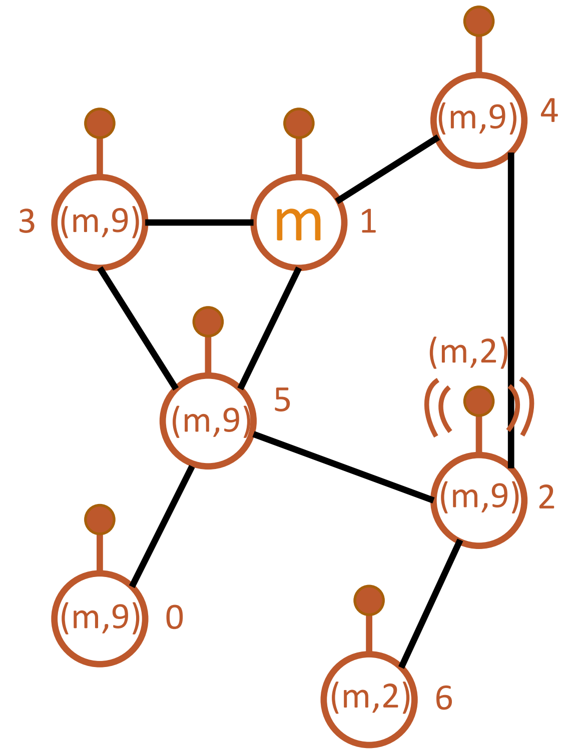
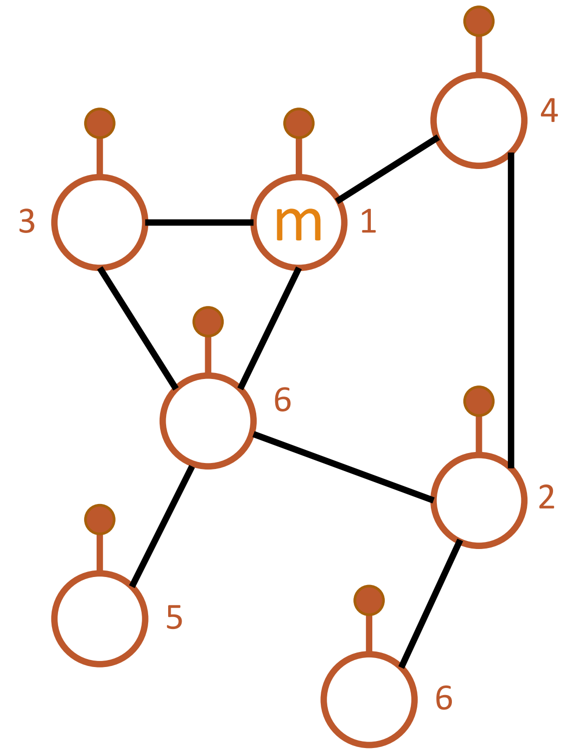


slot 9

DETERMINISTIC RADIO BROADCAST

- Initially, the source knows a source message m.
- Goal: all nodes eventually know m.

If each node has a distinct $(\log n + O(1))$-bit label, then $O(Dn)$ slots are sufficient.



D = source eccentricity
= $\max\{dist(source,v) \mid v \in V\}$

DETERMINISTIC RADIO BROADCAST ALGORTHM

Labelling scheme:

- Given a network G, assign a label to each node of G.

Transmission protocol:

- Each slot, each node determines, using only its label and its history, whether it should transmit, and, if so, what to transmit.

DETERMINISTIC RADIO BROADCAST ALGORITHMS

Chlamtac & Weinstein 1991

- $O(\log^2 n)$ slots for networks with $D = 2$.
- $O(D \log^2 n)$ slots for arbitrary networks.

⋮

Kowalski & Pelc 2007

- $D + O(\log^2 n)$ slots for arbitrary networks
- $D + O(\log^2 n)$ bits for each label:
  indicates the slots in which the node transmits.

LOWER BOUNDS FOR DETERMINISTIC RADIO BROADCAST

Alon, Bar-Noy, Linial & Peleg 1991

- $\Omega(D + \log^2 n)$ slots are necessary for some networks,
  no matter how large the labels are

DETERMINISTIC RADIO BROADCAST ALGORITHMS

Ellen, Gorain, Miller & Pelc 2019

- 2 bits for each label

  label of each node is carefully chosen based on knowledge of the entire network
- ≤ 2n-3 slots for arbitrary networks

Ellen & Gilbert 2020

- 4 bits for each label, $O((nD)^{1/2})$ slots

  The label of each node is carefully chosen based on the knowledge of the entire network.
- Defined continuous broadcast algorithms

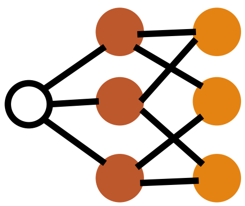
  Proved that any algorithm in this class requires $\Omega((nD)^{1/2})$ slots for some network

Ellen & Gilbert 2020

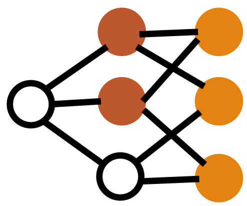
- 3 bits for each label, $O(D \log n + \log^2 n)$ slots
  The labels are chosen non-constructively.
- 3 bits for each label, $O(D \log^2 n)$ slots
  There is a deterministic algorithm for constructing labels.

DOMINATING SET

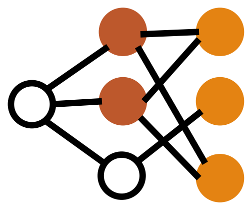
Let **X** and **Y** be sets of nodes in a network G.
X is a dominating set for **Y** if every node in **Y** is a neighbour of some node in **X** and
is minimal if no proper subset of **X** is a dominating set for **Y**.



X is a dominating set for **Y**



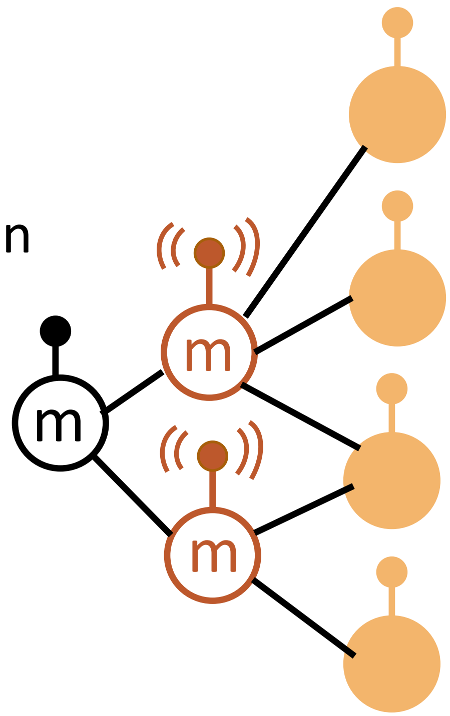
X is a minimal dominating set for **Y**



X is not a dominating set for **Y**

DOMINATING SET

Suppose all nodes in **X** know message m and
X is a minimal dominating set for **Y**.
If the nodes in **X** transmit m and
no other nodes transmit in the same slot, then
each node $u \in$ **X** has
a private neighbour $v \in$ **Y**
that receives m only from u.



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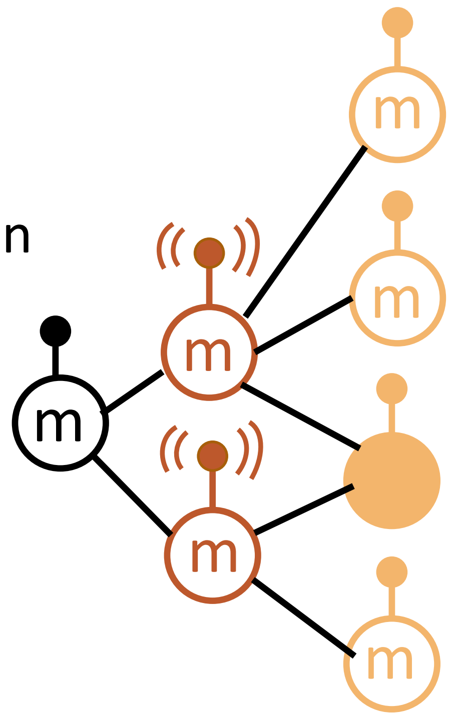


If $u \in$ **X** doesn't have a private neighbour,
then **X** – {u} is a dominating set for **Y**,
so **X** isn't a minimal dominating set for Y.

DOMINATING SET MECHANISM
Ellen, Gorain, Miller, & Pelc 2019

The label of each node consists of 2 bits:

- a JOIN bit and
- a STAY bit

and each round consists of 2 slots:

- a dominator slot and
- a feedback slot.

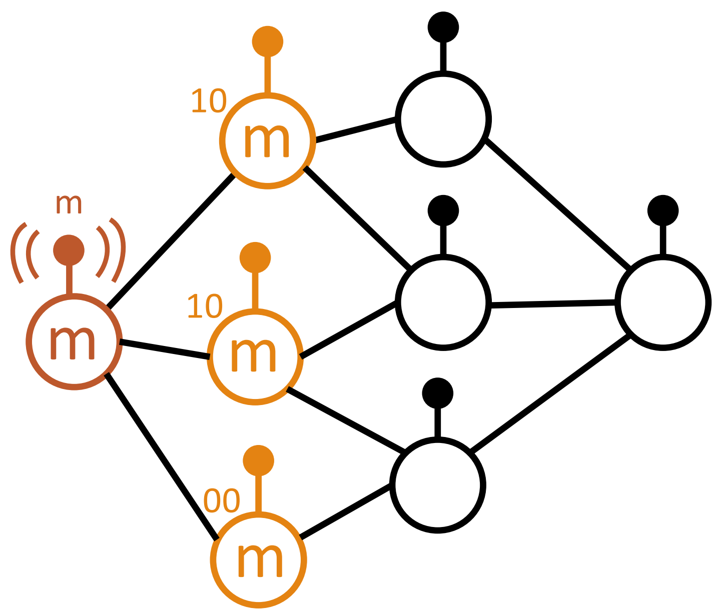
DOMINATING SET MECHANISM

- $INFORMED_r$ = the set of nodes that know the source message m at the beginning of round r
- $FRONTIER_r$ = the set of nodes not in $INFORMED_r$ that have at least one neighbour in $INFORMED_r$
- DOM_r = a subset of $INFORMED_r$ that is a minimal dominating set for $FRONTIER_r$

DOMINATING SET MECHANISM

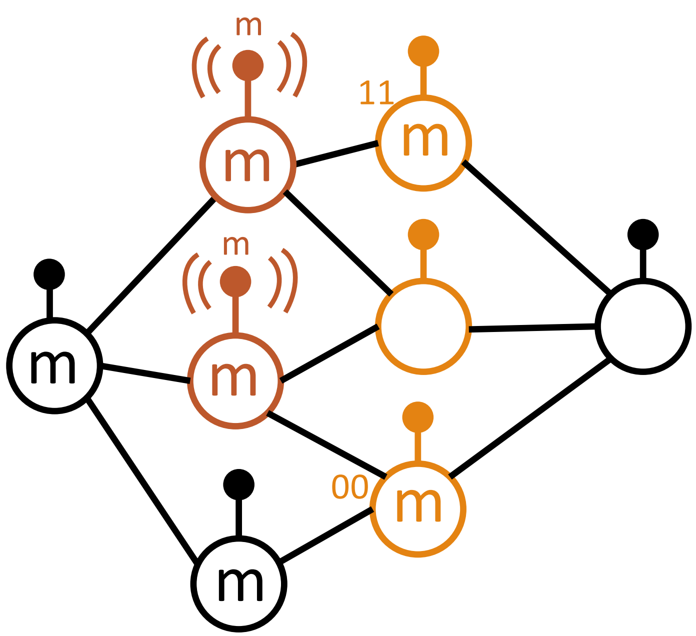
- In the dominator slot of round r,
  each node in DOM_r transmits the source message m.
- A newly informed node is in DOM_{r+1} if its JOIN bit is 1.
- Each node $u \in DOM_r$ has one private neighbour $v_u \in FRONTIER_r$
  as its designated feedback node.
  If the STAY bit of v_u is 1, then v_u transmits 1 in the feedback slot
  of round r and u is in DOM_{r+1}.

The JOIN and STAY bits are constructed by simulating the algorithm.



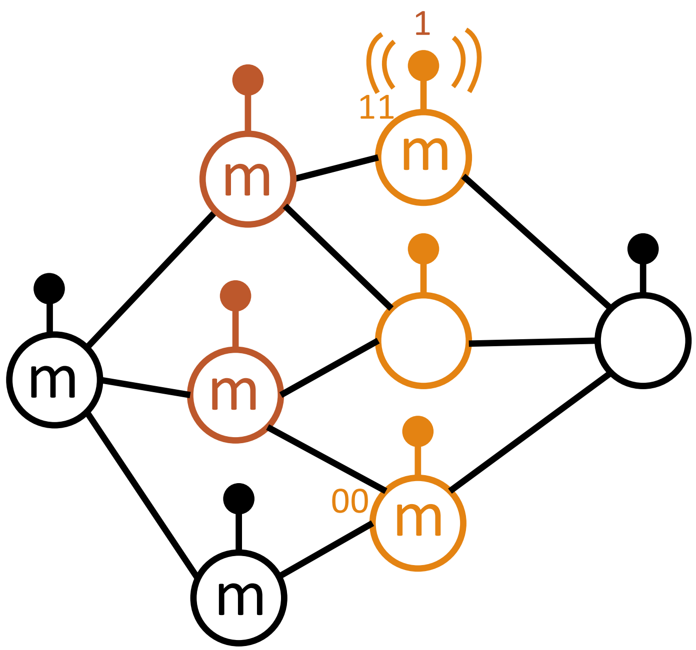
Round 1:

- DOM_1 transmits m in dominator slot.
- Each node in $FRONTIER_1$ receives the message and has STAY bit 0.
- No node transmits in the feedback slot.
- Two nodes in $FRONTIER_1$ have JOIN bit 1.



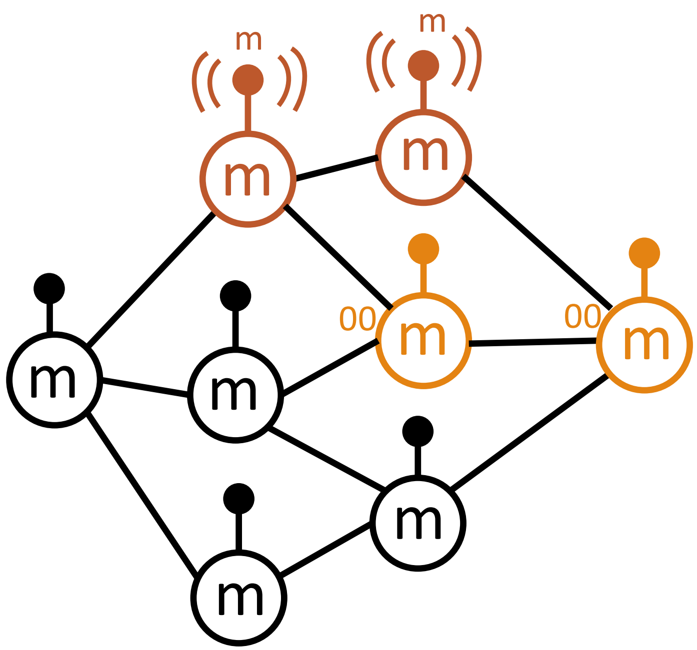
Round 2:

- DOM$_2$ transmits m in dominator slot.
- Two nodes in FRONTIER$_2$ receive the message.



Round 2:

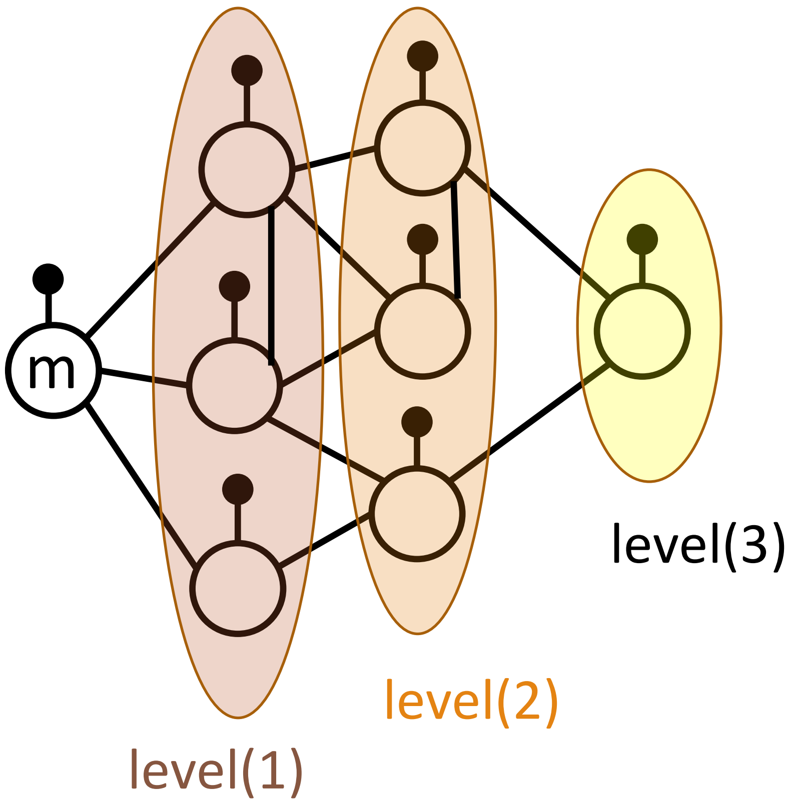
- DOM_2 transmits in dominator slot.
- Two nodes in $FRONTIER_2$ receive the message.
- One of them has STAY bit 1 and JOIN bit 1.
  It transmits in the feedback slot.



Round 3:

- DOM$_3$ transmits in dominator slot.
- Both nodes in FRONTIER$_3$ receive the message and have STAY bit 0 and JOIN bit 0.

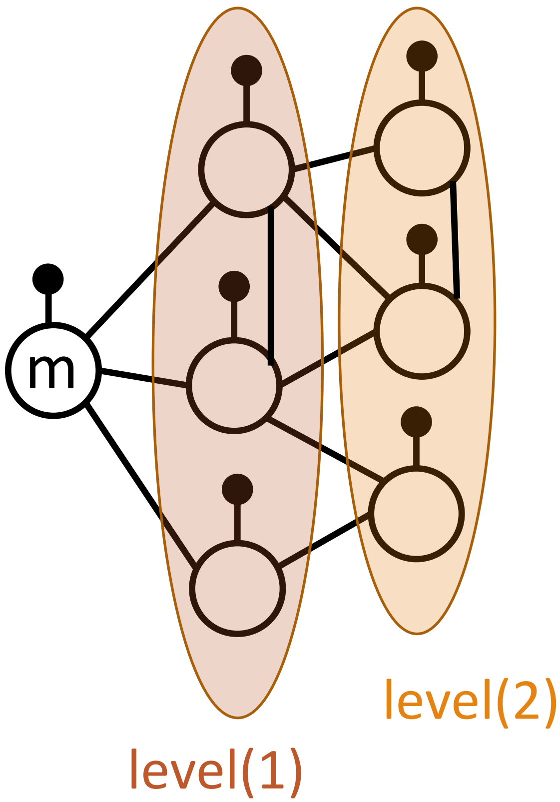
LEVELLED DOMINATING SET MECHANISM, Ellen & Gilbert 2020



level of a node = its distance from the source

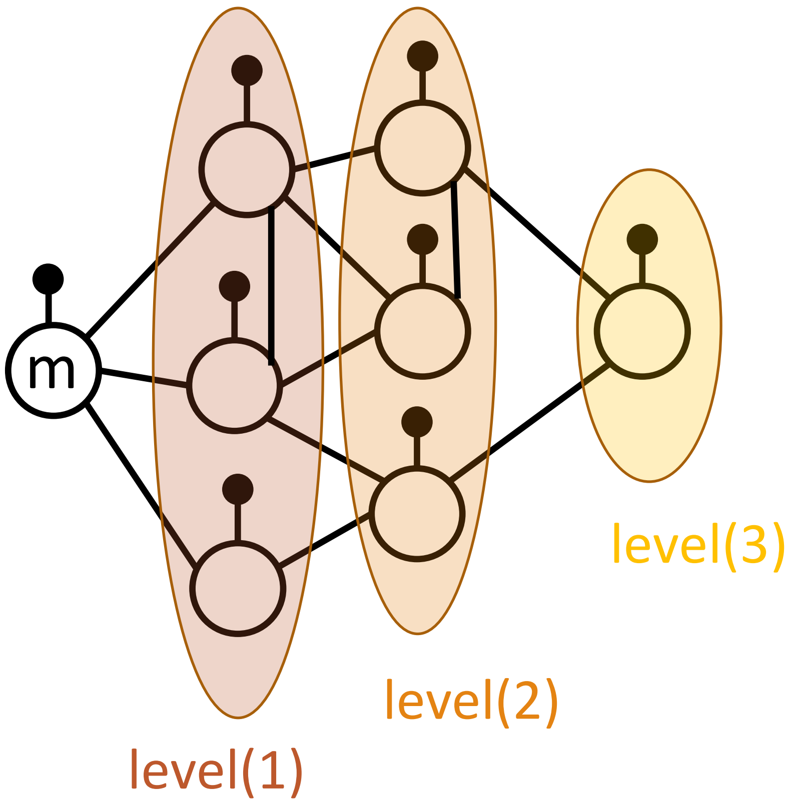
A node in level(L) only has neighbours in level(L-1), level(L), or level(L+1)

LEVELLED DOMINATING SET MECHANISM



For networks with source eccentricity 2,
x nodes at level 1, and
y nodes at level 2,
choose $DOM_{r+1} \subseteq DOM_r$ for $r \geq 2$.
Then the dominating set mechanism completes in $1 + \min\{\mathbf{x},(\mathbf{2y})^{1/2}\} \subseteq O(n^{1/2})$ rounds.

LEVELLED DOMINATING SET MECHANISM



level of a node = its distance from the source

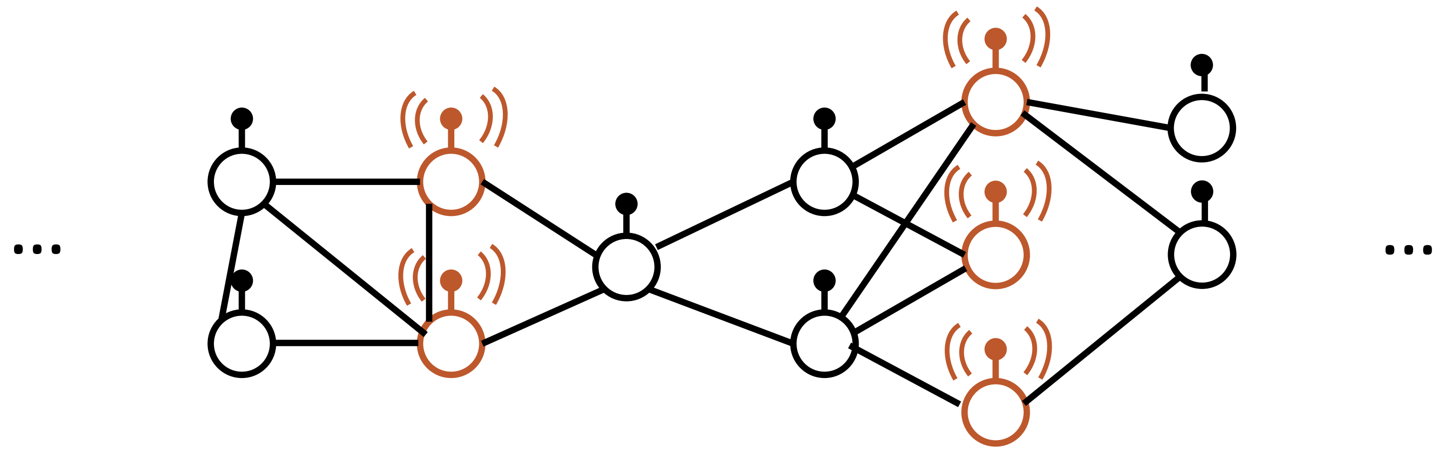
label of each node: JOIN bit, STAY bit, and its level mod 3 (2 bits)

LEVELLED DOMINATING SET MECHANISM

- Each node appends its level mod 3 when it transmits the source message m.
- A node in level(L) ignores the source message unless it comes from a node in level(L-1).
- $DOM_r \cap$ level(L-1) is a minimal dominating set for $FRONTIER_r \cap$ level(L).

LEVELLED DOMINATING SET MECHANISM

In round r, if a node in level(L-1) transmits m in the dominator slot or a node in level(L) transmits 1 in the feedback slot, then $L \equiv r \bmod 3$.

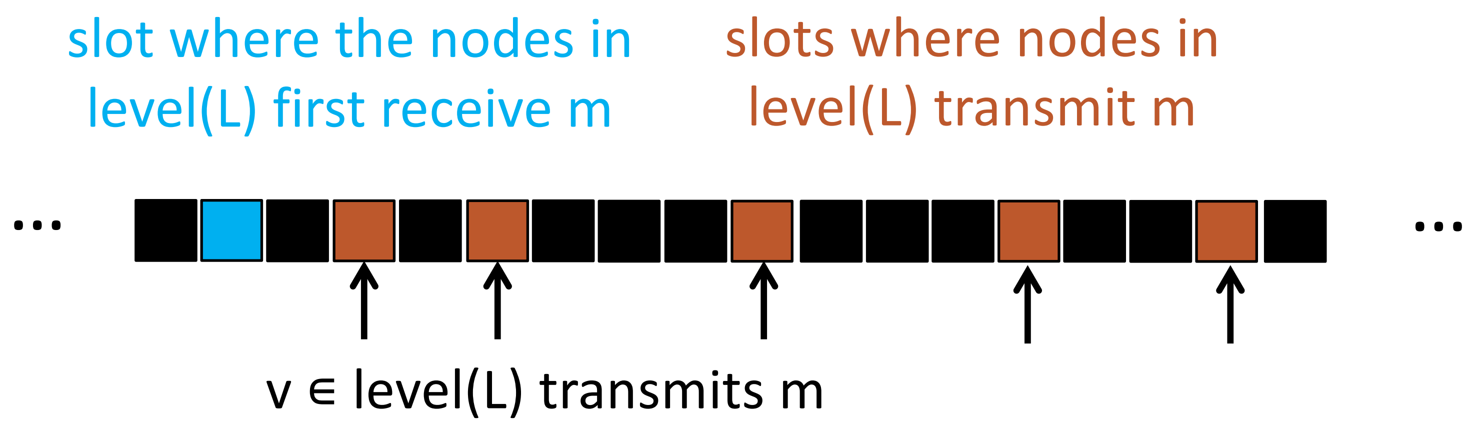


LEVELLED DOMINATING SET MECHANISM

THEOREM For any network with n nodes and source eccentricity D, the levelled dominating set mechanism performs broadcast in $O((nD)^{1/2})$ rounds.

CONTINUOUS BROADCAST ALGORITHM

Suppose all nodes in level(L) first receive m in the same slot.



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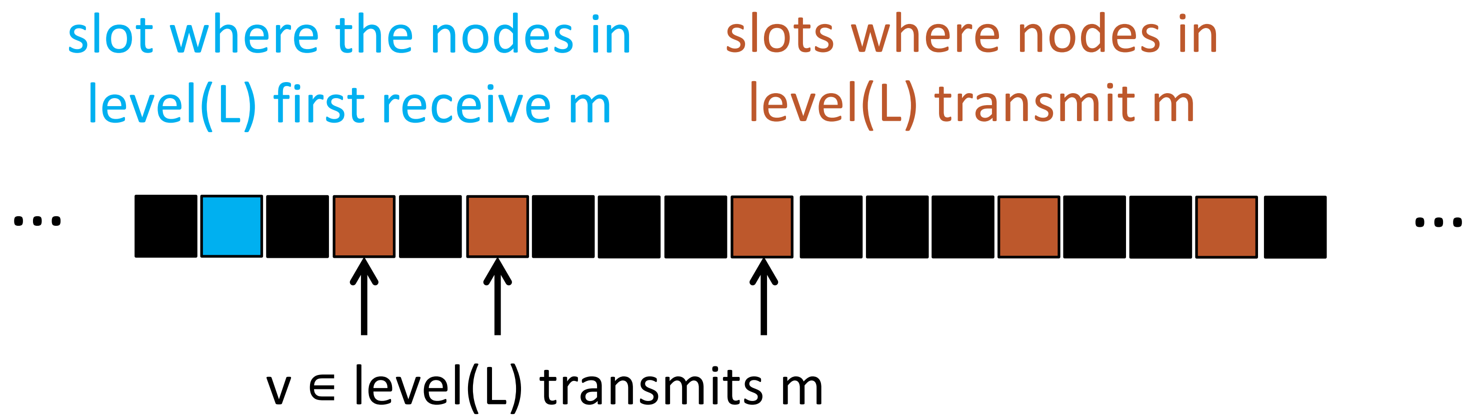

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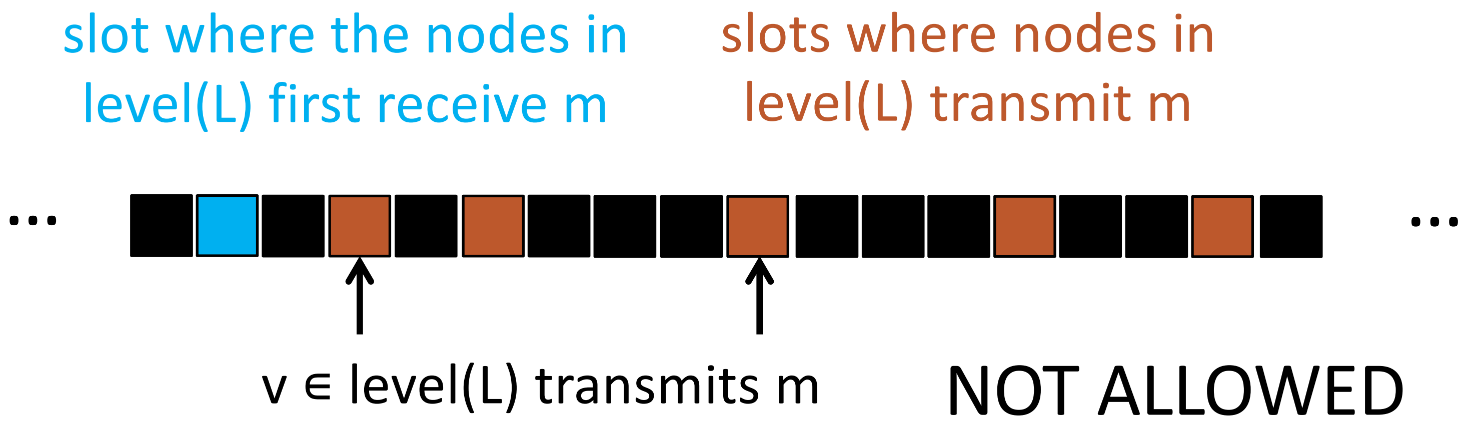
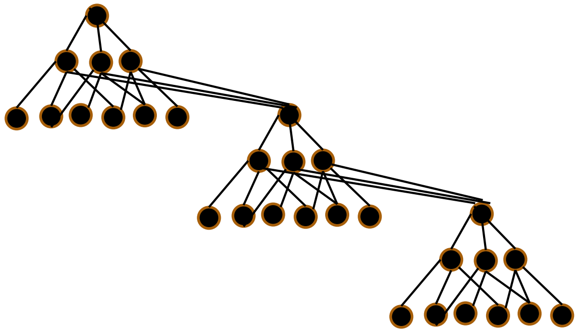


CONTINUOUS BROADCAST ALGORITHM

For every network and for every $L \geq 1$,
if all nodes in level(L) first receive the source message m in slot s,
then, for every node v in level(L) and all slots $s < s' < s''$,
if there is a node in level(L) that transmits m in slot s' and
v transmits m in slot s'', then
v transmits m in slot s'.

CONTINUOUS BROADCAST ALGORITHM

THEOREM For any continuous broadcast algorithm,
there is a network with n nodes and source eccentricity D that requires $\Omega((nD)^{1/2})$ slots.



PROPAGATION MECHANISM Ellen & Gilbert 2020

The label of each node consists of 3 bits:

- a JOIN bit,
- a STAY bit, and
- ➢ a GO bit

and each round consists of 3 slots:

- a dominator slot,
- a feedback slot, and
- ➢ a propagation slot.

PROPAGATION MECHANISM

- In the dominator slot of round r, each node in DOM_r transmits m.
- A newly informed node is in DOM_{r+1} if its JOIN bit is 1.
- Each node $u \in DOM_r$ has one private neighbour $v_u \in FRONTIER_r$ as its designated feedback node.
- ➢ v_u transmits its STAY and GO bits in the feedback slot of round r.
- If the STAY bit of v_u is 1, then u is in DOM_{r+1}.
- ➢ If the GO bit of v_u is 1, then u transmits the source message m in the propagation slot of round r.

The JOIN and STAY bits are constructed by simulating the algorithm.

CONSTRUCTING THE GO BITS PROBABILISTICALLY

- For each round r, independently choose $P(r) \in \{1,2,\ldots,\lceil \log_2 n \rceil\}$ uniformly.
- In round r, independently choose the GO bit of each designated feedback node to be
  1 with probability $2^{-P(r)}$ and
  0 with probability $1 - 2^{-P(r)}$.

CONSTRUCTING THE GO BITS PROBABILISTICALLY

LEMMA If v is in $FRONTIER_r$
then v receives the source message m in the propagation slot of round r
with probability $\geq 1/(30 \log_2 n)$.

CONSTRUCTING THE GO BITS PROBABILISTICALLY

LEMMA If v is in $FRONTIER_r$
then v receives the source message m in the propagation slot of round r
with probability $\geq 1/(30 \log_2 n)$.

Let d = number of neighbours of v in DOM_r .
First consider $d = 1$.
When $P(r) = 1$, the neighbour of v in DOM_r transmits with probability $1/2$.
Since $P(r) = 1$ with probability $1/\lceil \log_2 n \rceil$,
v receives the source message in the propagation slot of round r
with probability at least $1/(2 \lceil \log_2 n \rceil) > 1/(4 \log_2 n)$.

LEMMA If v is in $FRONTIER_r$
then v receives the source message m in the propagation slot of round r
with probability $\geq 1/(30 \log_2 n)$.

Let d = number of neighbours of v in DOM_r .
If each of these d neighbours transmits independently with probability q,
then the probability that exactly one of them transmits and, hence,
v receives the source message m in the propagation slot of round r
is $d\, q\, (1- q)^{d-1}$.
This is at least $1/15$ if $q = 2^{-P(r)}$ and $d \leq 2^{P(r)} \leq 2d$.
Since $d \leq 2^{P(r)} \leq 2d$ with probability at least $1/ \lceil \log_2 n \rceil$,
v receives the source message in the propagation slot of round r
with probability at least $1/(15 \lceil \log_2 n \rceil) > 1/(30 \log_2 n)$.

CONSTRUCTING THE GO BITS PROBABILISTICALLY

LEMMA If v is in $FRONTIER_r$
then v receives the source message m in the propagation slot of round r
with probability $\geq 1/(30\log_2 n)$.

LEMMA For any constant $c > 0$, a fixed node in level(L)
knows the source message m within $O(L \log n + \log^2 n)$ rounds
with probability $\geq 1 - n^{-c}$.

CONSTRUCTING THE GO BITS PROBABILISTICALLY

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LEMMA With high probability, every node
knows the source message m within $O(D \log n + \log^2 n)$ rounds.

CONSTRUCTING THE GO BITS PROBABILISTICALLY

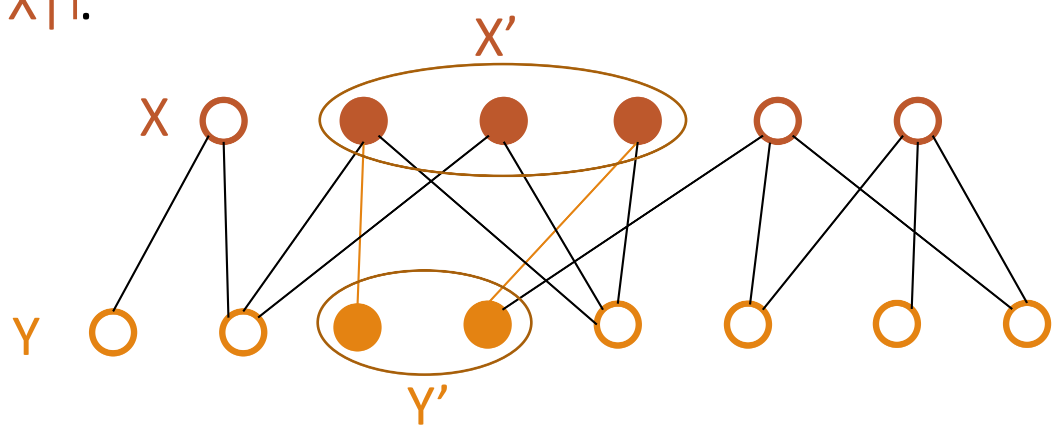
LEMMA With high probability, every node knows the source message m within $O(D \log n + \log^2 n)$ rounds.

THEOREM For any network, there is a choice for all the GO bits so that the propagation mechanism completes in $O(D \log n + \log^2 n)$ slots.

THEOREM [Alon, Bar-Noy, Linial & Peleg 1991]
$\Omega(D + \log^2 n)$ slots are necessary for some networks, no matter how large the labels are.

DETERMINISTIC RADIO BROADCAST FOR NETWORKS WITH SOURCE ECCENTRICITY 2 CHLAMTAC & WEINSTEIN 1991

LEMMA Given a connected bipartite graph with parts X and Y, there is an efficient algorithm to compute $X' \subseteq X$ and $Y' \subseteq Y$ such that every node in Y' has exactly one neighbour in X' and $|Y'| \geq |Y|/15\lceil \log_2|X| \rceil$.



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For $p = 1,\ldots,\lceil \log_2 |X| \rceil$, let $Y_p = \{ v \in Y \mid 2^{p-1} \leq \text{degree}(v) \leq 2^p \}$.
Let p be such that $|Y_p| \geq |Y|/\lceil \log_2 |X| \rceil$.

If each node in X transmits with probability 2^{-p}, then each node $v \in Y_p$ has probability $d2^{-p}(1- 2^{-p})^{d-1} \geq 1/15$ of receiving the message, where $d = \text{degree}(v)$.

Thus the expected number of nodes in Y that receive the message is at least $\mu = |Y|/15 \lceil \log_2|X| \rceil$.

If each node in X transmits with probability 2^{-p}, then the expected number of nodes in Y that receive a message is at least $\mu = |Y| / 15 \lceil \log_2 |X| \rceil$.

Initialize $X' = X'' = \emptyset$ and consider the nodes $u \in X$ one by one.

Experiment 1:
node u transmits
μ' = expected number of nodes in Y that receive a message

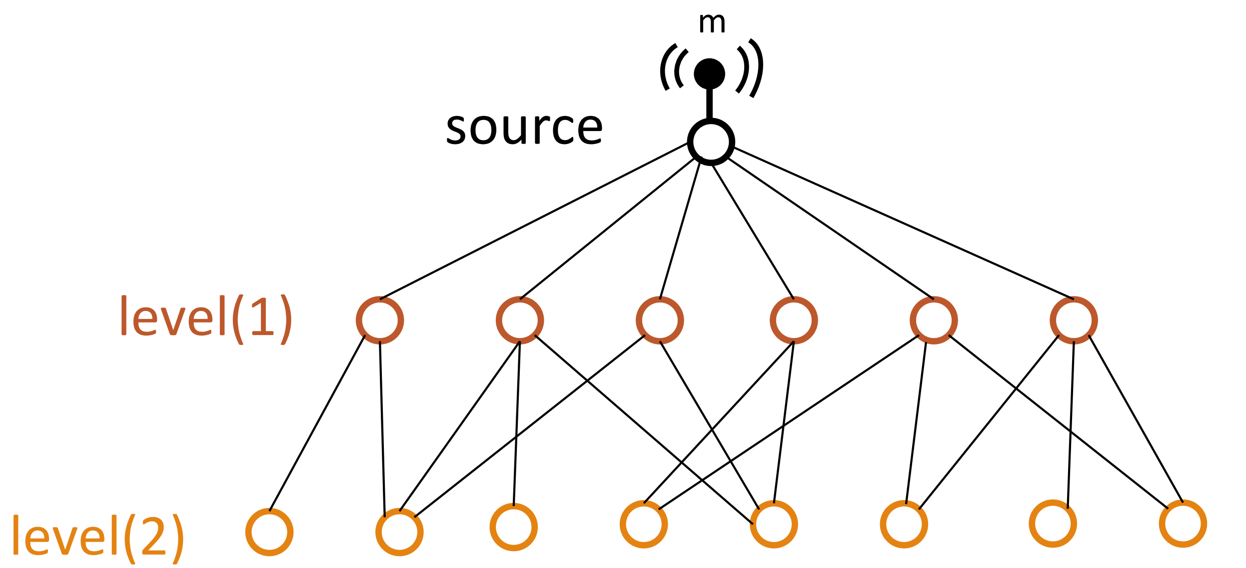
Experiment 2:
node u doesn't transmit
μ'' = expected number of nodes in Y that receive a message

Both experiments:
nodes in X' transmit
nodes in X'' don't transmit
nodes in $X - (X' \cup X'' \cup \{u\})$ each transmit with probability 2^{-p}

If $\mu' > \mu''$ add u to X'
else add u to X''.
Note: $\max\{\mu', \mu''\} \geq \mu$.

DETERMINISTIC RADIO BROADCAST FOR NETWORKS WITH SOURCE ECCENTRICITY 2 CHLAMTAC & WEINSTEIN 1991

round 1



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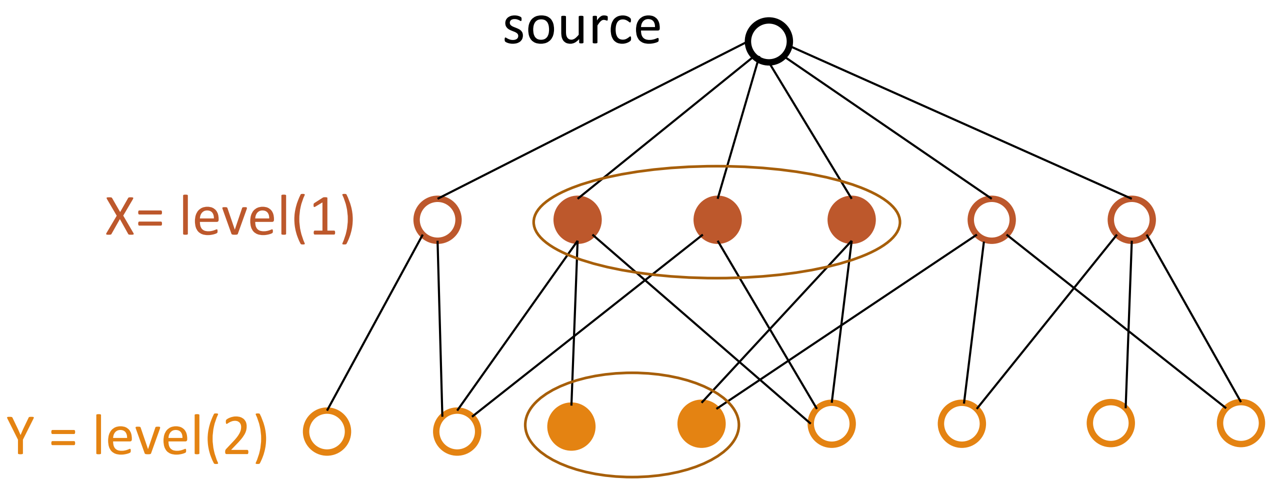
round 2



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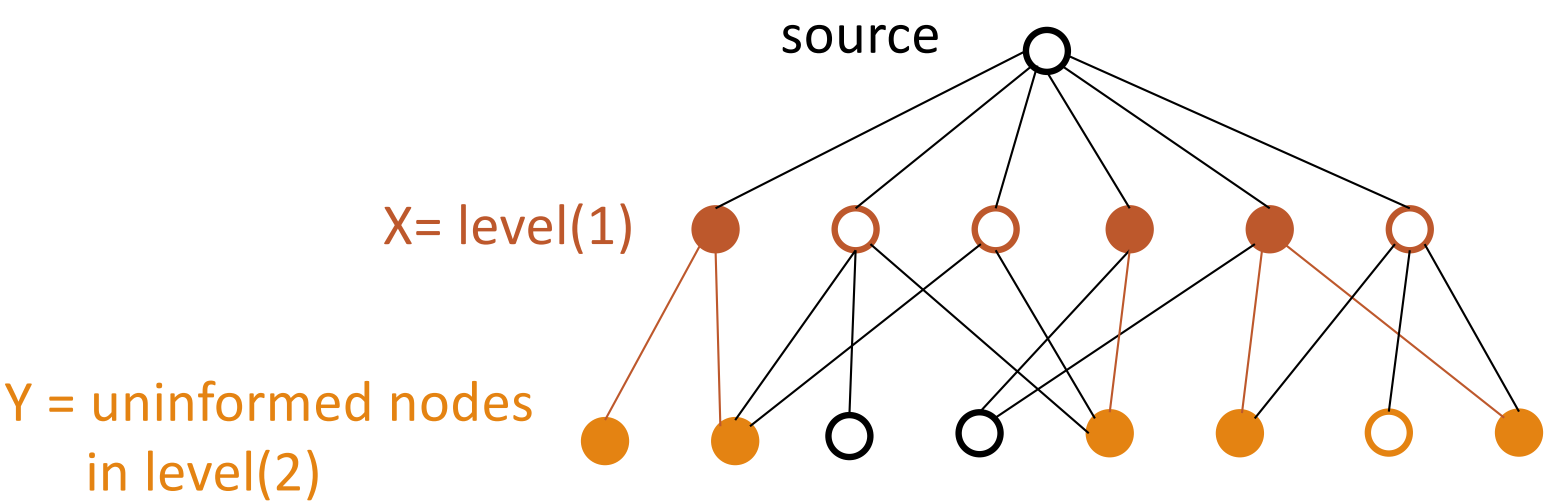
rounds $3, \ldots, 15 \lceil \log_2 n \rceil^2$



CONSTRUCTING THE GO BITS DETERMINISTICALLY

LEMMA Given a connected bipartite graph with parts X and Y, there is an efficient algorithm to compute $X' \subseteq X$ and $Y' \subseteq Y$ such that every node in Y' has exactly one neighbour in X' and $|Y'| \geq |Y|/15 \lceil \log_2 |X| \rceil$.

At each round r, let $L = \min\{k \mid FRONTIER_r \cap level(k) \neq \phi\}$, $Y = FRONTIER_r \cap level(L)$ and $X = DOM_r \cap level(L-1)$.
The nodes in X' transmit the source message m in the propagation slot and the GO bits of their designated feedback nodes are 1.

CONSTRUCTING THE GO BITS DETERMINISTICALLY

LEMMA If every node in level(L-1) knows the source message m by round r,
then every node in level(L) knows the source message m by round $r + 15 (\log_2 n)^2$.

THEOREM Every node knows the source message m within $O(D \log^2 n)$ slots.

THEOREM For any network, there exists a choice for the GO bits so that every node knows m within $O(D \log n + \log^2 n)$ slots.

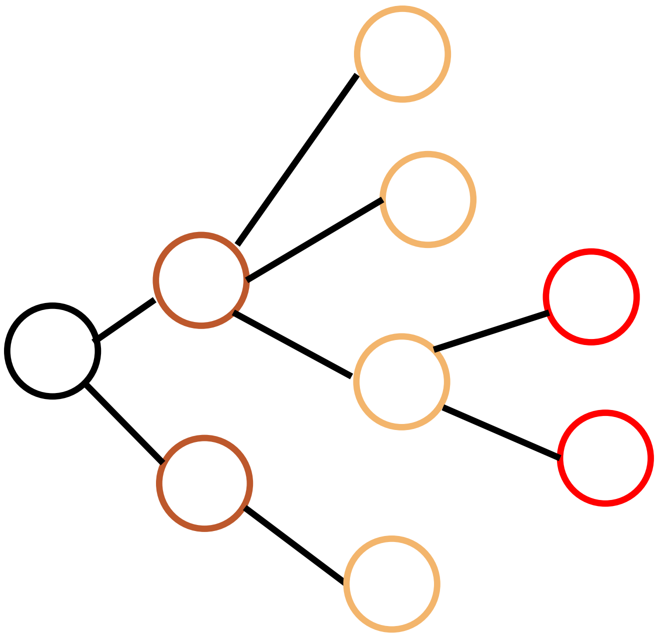
THEOREM For any network, one can deterministically construct the GO bits so that every node knows m within $O(D \log^2 n)$ slots.

THEOREM [Alon, Bar-Noy, Linial & Peleg 1991]
$\Omega(D + \log^2 n)$ slots are necessary for some networks, no matter how large the labels are.

OPEN QUESTION
Is there a deterministic radio broadcast algorithm with $O(1)$-bit labels that completes within $O(D + \log^2 n)$ slots?

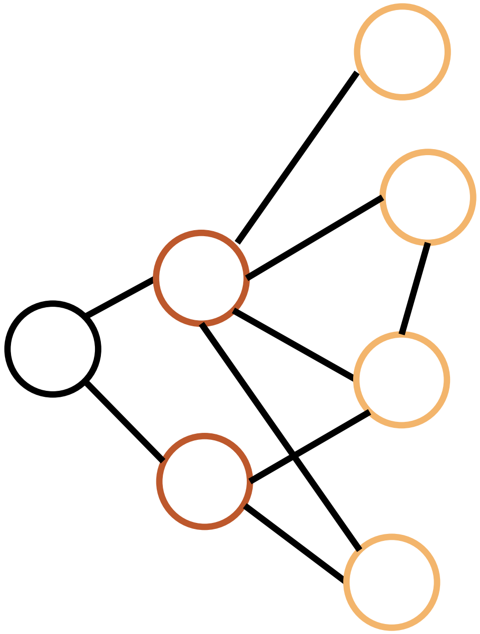
LABELLING SCHEMES WITH SMALLER LABELS FOR SPECIFIC FAMILIES OF NETWORKS

In trees, deterministic radio broadcast can be performed without labels:
Each process transmits the source message the slot after it receives it.



In networks with source eccentricity $D = 1$, deterministic radio broadcast can be performed without labels.

In networks with source eccentricity $D = 2$, using the levelled dominating set mechanism, all nodes at level 1 have STAY = 0 and all nodes at level 2 have JOIN = 0, so 1-bit labels suffice, provided a node learns its level when it first receives the source message.



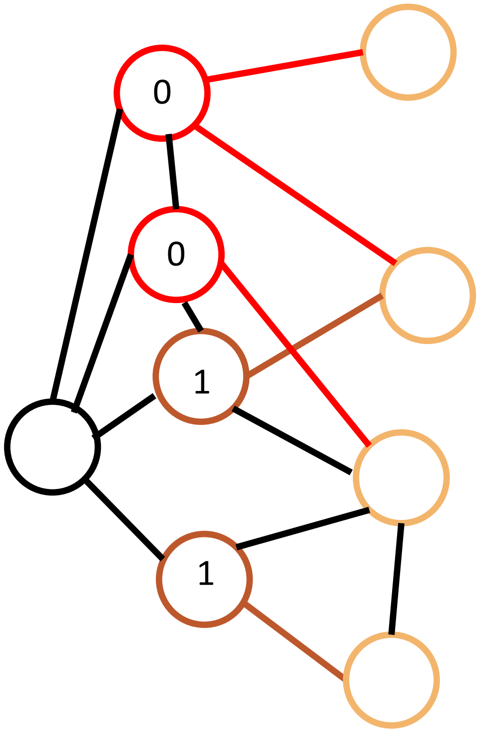
LEVEL SEPARABLE NETWORKS
BU, POTOP-BUTUCARU & RABIE 2020

A network with a distinguished source node is level-separable if, for every level $1 \leq k < D$, the nodes at level k can be partitioned into 2 parts such that every node at level $k+1$ has exactly one neighbour in at least one of the parts.

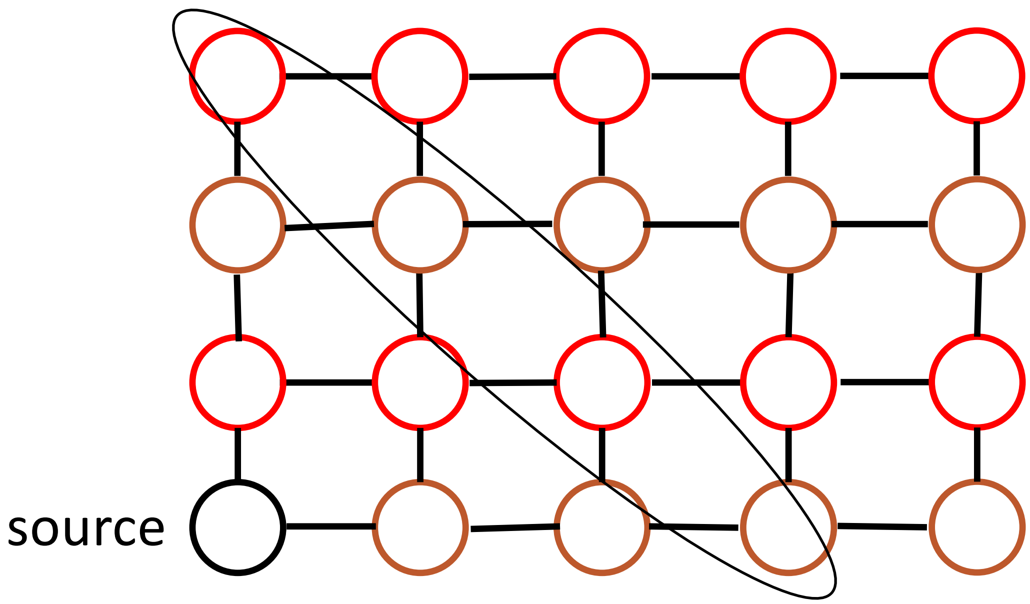
Each node has a 1-bit label, indicating which part of the level it belongs to.

In round $r > 1$, nodes in level $r-1$ transmit. The round consists of 2 slots, one for each of the 2 parts to transmit.

source eccentricity $D = 2$



A graph with a distinguished source node is level-separable if, for every level $1 \le k < D$, the nodes at level k can be partitioned into 2 parts such that every node at level $k+1$ has exactly 1 neighbour in at least one of the parts.



A two-dimensional grid is an example of a level-separable graph

THEOREM Deterministic radio broadcast cannot be performed without labels in a 4-cycle.

THEOREM For any network, deterministic radio broadcast be performed with 2-bit labels.

OPEN QUESTION

Can deterministic radio broadcast be performed with 1-bit labels in every network?

If not, characterize the family of networks for which deterministic radio broadcast can be performed with 1-bit labels.

ACKNOWLEDGED DETERMINISTIC RADIO BROADCAST

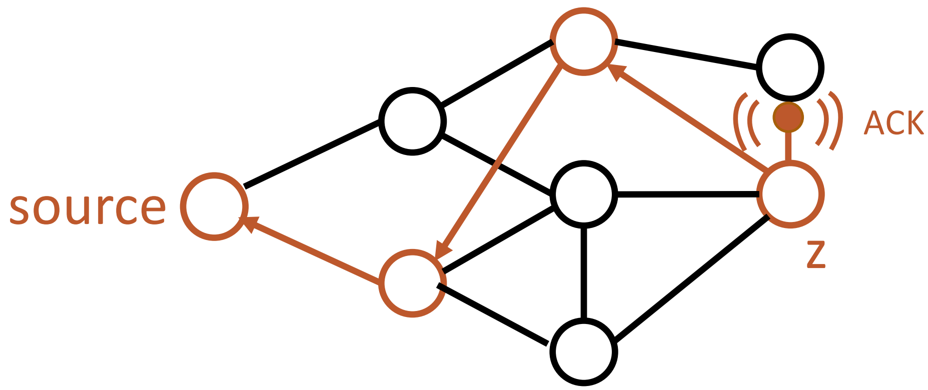
The source receives an acknowledgement message in some slot after all nodes receive the source message.

Can be accomplished using an EXTRA bit per label, determined as follows:

- Let z be a node that is informed last during the broadcast algorithm. Its EXTRA bit is 1. Note that its JOIN and STAY bits are 0.
- Consider a shortest path between the source and z in which all nodes except the endpoints have JOIN bit 1. Their EXTRA bits are 1.
- All nodes not on this path have EXTRA bit 0.

ACKNOWLEDGED DETERMINISTIC RADIO BROADCAST

When z is first informed, it transmits ACK in the next slot.
When any other node with EXTRA bit 1 first receives ACK,
it transmits ACK in the next slot.



DETERMINISTIC RADIO BROADCAST FROM AN ARBITRARY SOURCE

Suppose the source node for the broadcast is chosen after the nodes of the network have been labelled.

This version of deterministic radio broadcast can performed using a variant of acknowledged broadcast, with the same number of bits per label as acknowledged deterministic radio broadcast.

A surrogate source broadcasts a request for a message.
The actual source acknowledges with the source message m.
Then the surrogate source broadcasts m.

ACKNOWLEDGED BROADCAST VERSION 2

When node z receives m, it transmits an ACK message, which will travel back to the source along the same path that m took from the source to z.

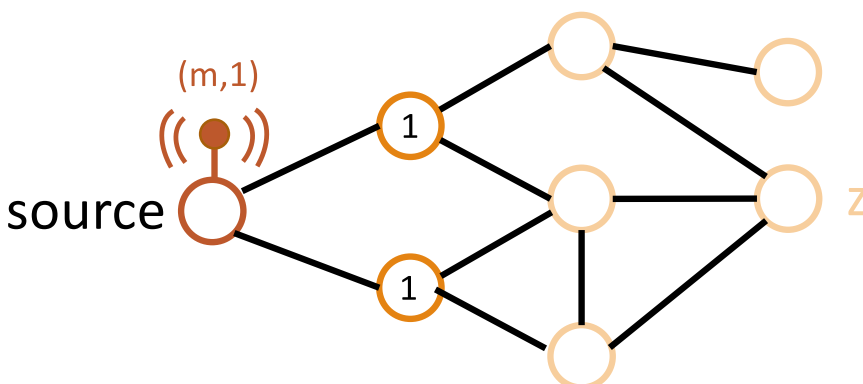
The unique node z_1 that informed z will transmit ACK when it receives ACK.

The unique node z_2 that informed z_1 will transmit ACK when it receives ACK.

:

ACKNOWLEDGED BROADCAST VERSION 2

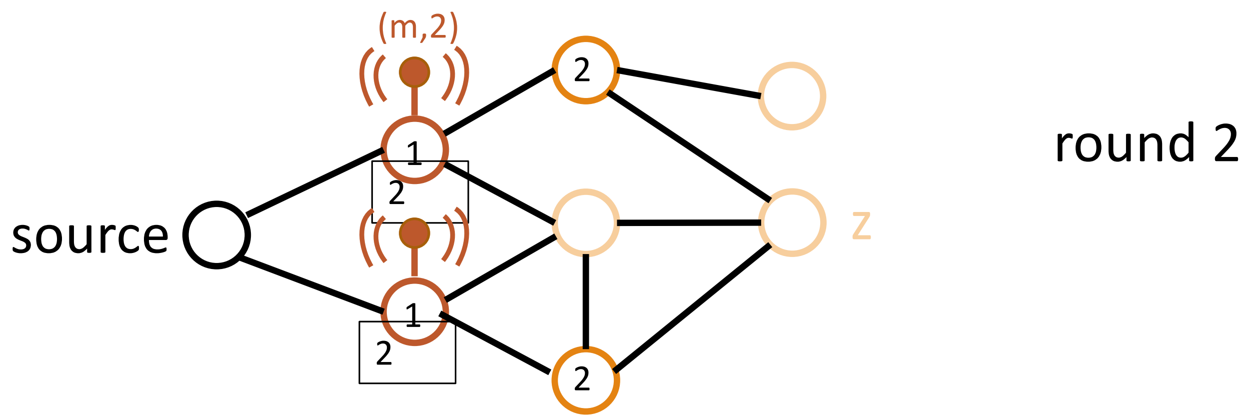
The source sends a counter c = 1 along with m.
When a node v first receives m, it stores the value of c.
It also sets a local counter to c, which it increments each round.
When v transmits m, it sends the value of its local counter along with m and keeps track of the values of its local counter that it has sent.



round 1

ACKNOWLEDGED BROADCAST VERSION 2

To facilitate this, the source sends a counter c = 0 along with m.
When a node v first receives m, it stores the value of c.
Node v also sets a local counter to c, which it increments each round.
When v transmits m, it sends the value of its local counter along with m and keeps track of the values of its local counter it has sent.



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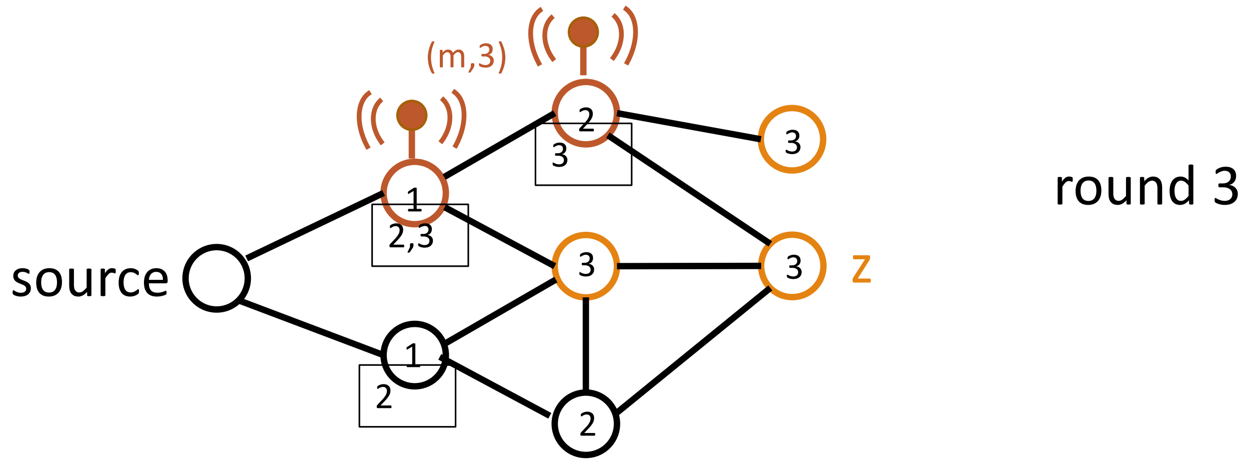
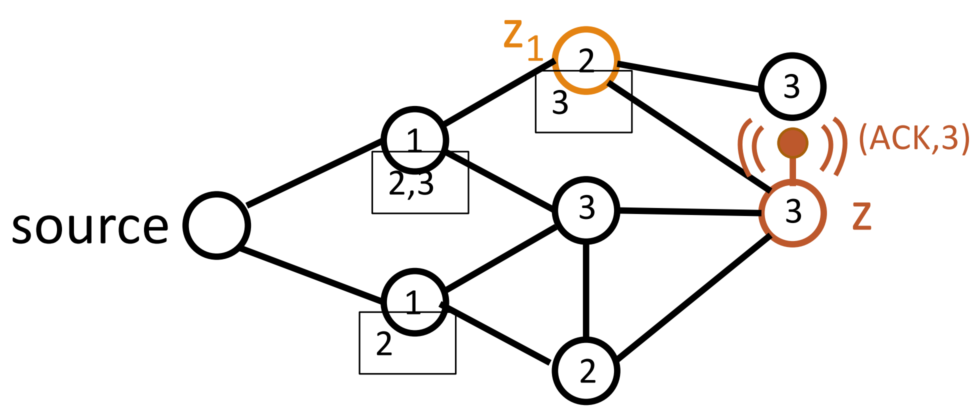


ACKNOWLEDGED BROADCAST VERSION 2

The round after z first receives m, it transmits ACK together with the counter value when it first received m.

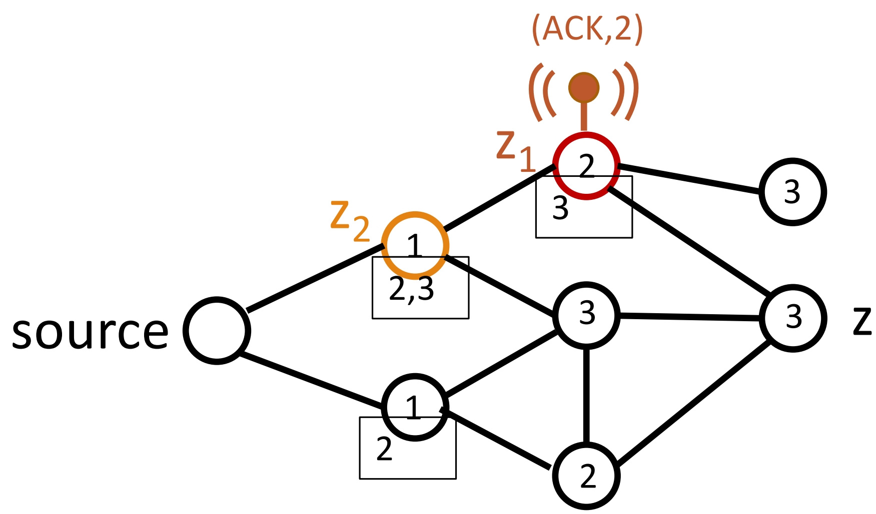
Only one of z's neighbours, z_1, transmitted that counter value.



round 4

ACKNOWLEDGED BROADCAST VERSION 2

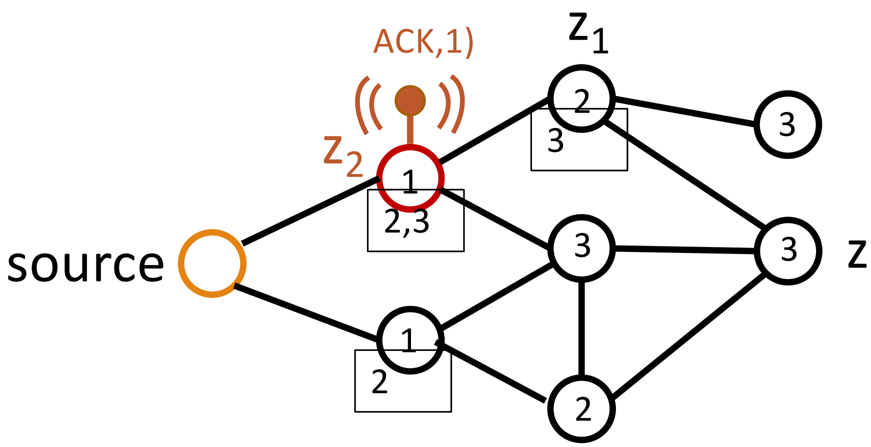
Only one of z's neighbours, z_1, transmitted that counter value.
z_1 forwards ACK together with the counter value when it first received m.
Only one of z_1's neighbours, z_2, transmitted that counter value.



round 5

ACKNOWLEDGED BROADCAST VERSION 2

Only one of z_1's neighbours, z_2, transmitted that counter value.
z_2 forwards ACK together with the counter value when it first received m.



round 6

OPEN QUESTIONS

Is an extra bit needed for acknowledged deterministic radio broadcast?

Is an extra bit needed for deterministic radio broadcast from an arbitrary source?